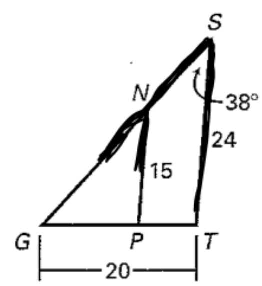
6.3

Similar Triangles

Example 1 *Writing Proportionality Statements*

In the diagram, $\triangle GST \sim \triangle GNP$.



a. Write the statement of proportionality.

b. Find $m\angle GNP$.

c. Find GP.

Solution

a. $\frac{GN}{\boxed{GS}} = \frac{GP}{\boxed{GT}} = \frac{NP}{\boxed{ST}}$

b. $\angle NST \cong \angle GNP$, so $m\angle GNP = \underline{38}°$.

c. $\frac{NP}{\boxed{ST}} = \frac{GP}{\boxed{GT}}$ **Write proportion.**

$\frac{15}{\boxed{24}} = \frac{GP}{\boxed{20}}$ **Substitute.**

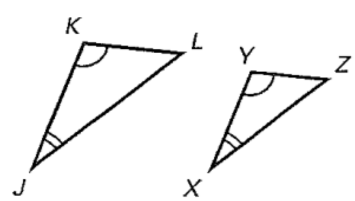
$\frac{\boxed{20}(15)}{\boxed{24}} = GP$ **Multiply each side by $\underline{20}$.**

$\underline{12.5} = GP$ **Simplify.**

Answer So, GP is $\underline{12.5}$ units.

POSTULATE 25: ANGLE-ANGLE (AA) SIMILARITY POSTULATE

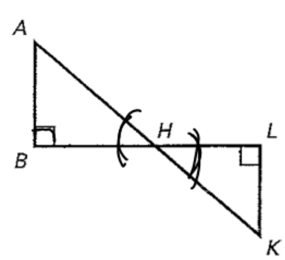
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



If $\angle JKL \cong \angle XYZ$ and $\angle KJL \cong \angle YXZ$, then $\triangle \underline{JKL} \sim \triangle \underline{XYZ}$.

Example 2 *Proving that Two Triangles are Similar*

In the diagram, $\triangle ABH \sim \triangle KLH$. Use properties of similar triangles to explain why these triangles are similar.

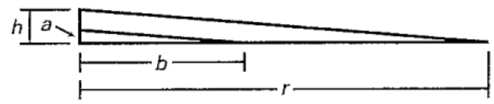


Solution

You can use the Vertical Angles Theorem to determine $\angle AHB \cong \angle \underline{KHL}$. Because they are right angles, $\angle ABH \cong \angle \underline{KLH}$. By the $\underline{\text{AA Similarity post}}$, you can conclude that $\triangle ABH \sim \triangle KLH$.

Example 3 *Using Similar Triangles*

To comply with the Americans with Disabilities Act, wheelchair ramps made for new constructions must have a height to length ratio of 1 : 12. At a new construction, the height h to the bottom of a door is 2.5 feet. Use the proportion $\frac{a}{b} = \frac{h}{r}$ to estimate the length r that the ramp should be to have the correct slope ratio. In the proportion, use $a = 1$ ft and $b = 12$ ft.



Solution

$$\frac{a}{b} = \frac{h}{r}$$ **Write proportion.**

$$\frac{\boxed{1}}{\boxed{12}} = \frac{\boxed{2.5}}{r}$$ **Substitute.**

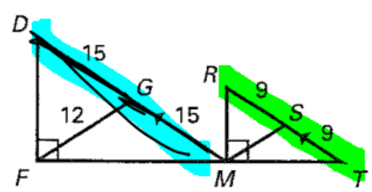
$r =$ 30 ft **Cross product property**

Answer The ramp should have a length of 30 feet.

Example 4 *Using Scale Factors*

Find the length of $\overline{MS}$.

First, find the scale factor of $\triangle DFM$ to $\triangle RMT$.



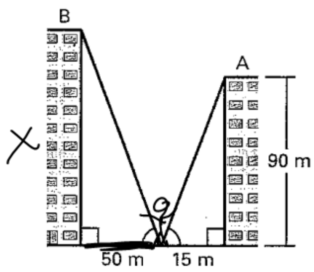
$$\frac{DM}{RT} = \frac{\boxed{15} + \boxed{15}}{\boxed{9} + \boxed{9}} = \frac{\boxed{30}}{\boxed{18}} = \frac{5}{3}$$

Now, because the ratio of the lengths of the segments is equal to the scale factor, you can write the following equation.

$$\frac{FG}{MS} = \frac{5}{3} \qquad \frac{12}{MS} = \frac{5}{3}$$

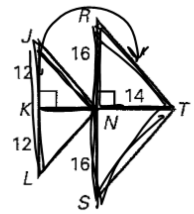
Answer Substitute 12 for FG and solve for MS to show that $MS =$ 7.2

1. You are standing 15 m from building A and 50 m from building B. Building A is 90 m tall. Find the height of building B.



$$\frac{15}{50} = \frac{90}{x}$$

2. $\triangle JNL \sim \triangle RTS$. Find the length of $\overline{KN}$.



$$\frac{JL}{RS} = \frac{12+12}{16+16} = \frac{24}{32} = \frac{3}{4}$$

$$\frac{3}{4} = \frac{KN}{14} \rightarrow KN = 10.5$$

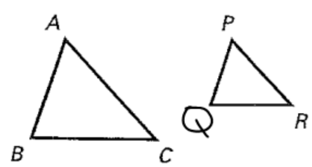
Proving Triangles are Similar

THEOREM 8.2: SIDE-SIDE-SIDE (SSS) SIMILARITY THEOREM

If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.

If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$,

then $\triangle ABC \sim \triangle PQR$.

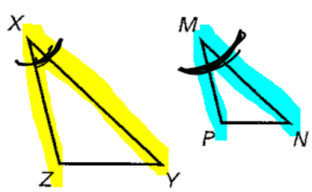


THEOREM 8.3: SIDE-ANGLE-SIDE (SAS) SIMILARITY THEOREM

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

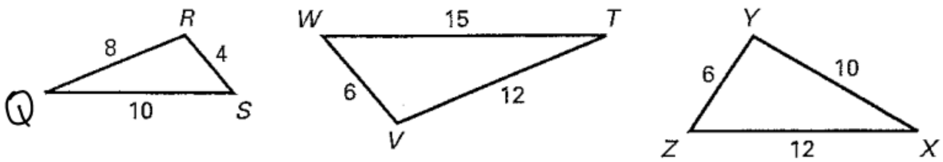
If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$,

then $\triangle ZXY \sim \triangle PMN$.



Example 1 *Using the SSS Similarity Theorem*

Which of the following three triangles are similar?



To decide which, if any, of the triangles are similar, you need to consider the ratios of the lengths of corresponding sides.

Ratios of Side Lengths of $\triangle QRS$ and $\triangle TVW$

Shortest sides $\frac{RS}{VW} = \frac{4}{6} = \frac{2}{3}$,

Longest sides $\frac{QS}{TW} = \frac{10}{15} = \frac{2}{3}$,

Remaining sides $\frac{QR}{TV} = \frac{8}{12} = \frac{2}{3}$

Answer Because the ratios are equal, $\triangle QRS \sim \triangle TVW$.

Ratios of Side Lengths of $\triangle QRS$ and $\triangle XYZ$

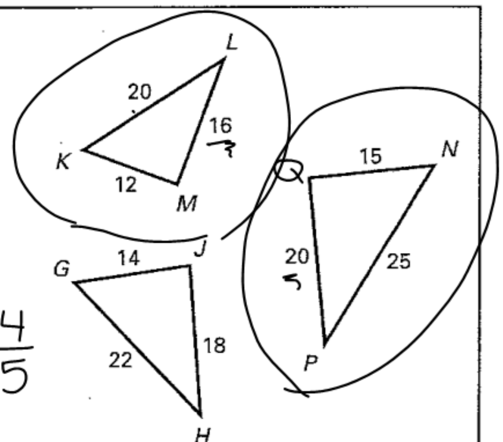
Shortest sides $\frac{RS}{YZ} = \frac{4}{6} = \frac{2}{3}$,

Longest sides $\frac{QS}{XZ} = \frac{10}{12} = \frac{5}{6}$,

Remaining sides $\frac{QR}{XY} = \_\_\_ = \_\_\_$

Answer Because the ratios are not equal, they are not similar ___________.

1. Which of the three triangles are similar?



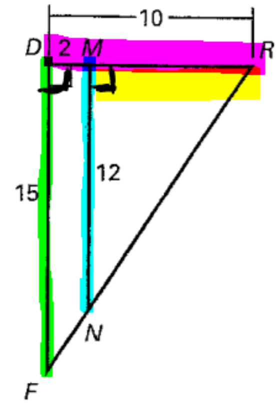
$\frac{12}{14} = \frac{6}{7}$ $\frac{20}{22} = \frac{10}{11}$

$\frac{12}{15} = \frac{4}{5}$ $\frac{20}{25} = \frac{4}{5}$ $\frac{16}{20} = \frac{4}{5}$

$\triangle KML \sim \triangle NQP$

Example 2 *Using the SAS Similarity Theorem*

Use the given lengths to prove that $\triangle DFR \sim \triangle MNR$.



Solution

Given: $DF = 15$, $MN = 12$
$DM = 2$, $DR = 10$

Prove: $\triangle DFR \sim \triangle MNR$

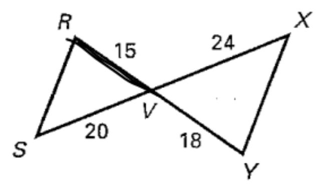
Paragraph Proof Use the SAS Similarity Theorem. Begin by finding the ratios of the lengths of the corresponding sides.

$\frac{DF}{MN} = \frac{15}{12} = \frac{5}{4}$

$\frac{DR}{MR} = \frac{10}{8} = \frac{5}{4}$

So, the lengths of sides DF and DR are proportonel to the lengths of the corresponding sides of $\triangle MNR$. Because $\angle FDR$ and $\angle NMR$ are congruent, use the SAS Similanty Theorem to conclude that $\triangle DFR \sim \triangle MNR$.

2. Describe how to prove that $\triangle RSV$ is similar to $\triangle YXV$.



$\frac{RV}{YV} = \frac{15}{18} = \frac{5}{6}$

$\frac{SV}{XV} = \frac{20}{24} = \frac{5}{6}$

$\angle RVS \cong \angle YVX$

$\rightarrow$ VA $\cong$

By SAS Similanty
$\triangle RSV \sim \triangle YXV$